

Theoretical Analysis of the Acceleration of Runaway Electron within the Earth's Atmosphere

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ABSTRACT

The acceleration of runaway electron within the Earth's atmosphere is the focus of this work. And we based our concentration on the use of mathematical analysis to examine the focus of this work within the non-relativistic region of the earth's atmosphere. This is done by using the equation of a cold electron. This equation is examined with consideration to the frictional force which the electrons undergo in the presence of electric field in the atmosphere. The effect of force on these electrons is also examined. The result shows that the rate of collisions of the electrons increases as the velocity decreases. And this could slow down some atmospheric processes within the non-relativistic region of the atmosphere. Hence, the spontaneous processes may not adequately occur within this region of the Earth's atmosphere.

INTRODUCTION

Runaway electron is observed in the outer radiation belt and at the Earth's magnetic bow shock, (Vasyliunas, 1980). These electrons can be generated in lightning path. This path can be monitored and its radiation bursts during lightning storms observed, (Tsuchiya et al, 2007). According to Gurevich, the theory of lightning initiation is known as the "runaway breakdown". This theory states that the energy inside a thundercloud, that force of positive and negative particles, is too weak to generate a spark to initiate lightning (SADE, 2009). This implies that for lightning to occur, the thundercloud must be struck by outside particles. These outside particles are burst of electrons that carry very high energy. With this added energy, a spark can be generated to initiate lightning. Though, these particles are from cosmic rays (Gurevich, 1992). This suggests that lightning strokes are triggered by cosmic rays which ionize atoms, releasing electrons that are accelerated by the electric fields, ionizing other air molecules and making the air conductive by a runaway breakdown, (Gurevich, 1992; Shrope, 2004). All the same, in the atmosphere, electrons and ions with charge, undergo an accelerating force from any electric field E present (Alexander, 2011).

When a weak electric field is applied to the atmosphere, the electron distribution develops a drift, a slight distortion and at very high energies, a runaway electron tail. The high energy tail extends to infinite momentum (or rather grows indefinitely with time) (Kruskal and Bernstein, 1962). The waves that interact with the runaways do not produce significant radial diffusion so that with formed magnetic surfaces, it is unlikely that the radial loss of runaways determines the steady state

(Molvig and Tekula, 1977).

In a nut shell, runaway electrons are accelerated by quasi-electrostatic fields in the middle atmosphere (Bell et al., 1995). This causes the runaway electron breakdown being produced by electrons from a cosmic ray shower (McCarthy and Parks, 1992). Runaway electron breakdown is an avalanche multiplication process proposed to occur in moderate electric fields in gases. Electron breakdown multiplication occurs whenever the electric field exceeds the threshold for runaway relativistic electron breakdown but cannot begin without the presence of the cause of relativistic electrons, requiring at least one energetic particle to initiate the process. Under normal conditions, the Earth's atmosphere has many such energetic particles resulting partly from radioactive decay but largely from cosmic rays (CRs) and the extensive air showers (EASs) of secondary particles they produce (Nikolai, 2000).

MATERIALS AND METHOD

The rate of acceleration of runaway electron within the non-relativistic region of the earth's atmosphere will be determined using an equation of a cold electron. This equation will be examined. And further application of some mathematical tools such as power series, and binomial expansion will clearly show the rate at which the electron velocity can accelerate within the non-relativistic region of the atmosphere with respect to the speed of light.

RESULT AND DISCUSSION

According to Milikh, a cold electron with mean velocity will have a dynamical frictional force given by;

$$F = mV_o v \quad (1)$$

Where,

$$V_o = \frac{4\pi e^4 Z N_m}{m^2 v^3} \log_e \Lambda$$

$$F = \frac{4\pi e^4 Z N_m}{m v^2} \log_e \Lambda \quad (2)$$

Where, $\log_e \Lambda = \log_e \left\{ \left(\frac{mc^2}{\epsilon_1} \right) \gamma \right\}$, γ is the Lorentz factor.

Considering a reduced dynamical frictional force of a fast electron with an increased velocity, the force will be,

$$F = \frac{4\pi e^4 Z N_m}{m c^2} \log_e \left\{ \left(\frac{mc^2}{\epsilon_1} \right) \gamma \right\} \quad (3)$$

$$\frac{F m c^2}{4\pi e^4 Z N_m} = \log_e \left(\frac{mc^2}{\epsilon_1} \right) \gamma \quad (4)$$

Multiply the left hand side of the equation (4) by E/E

$$\frac{E m c^2 F}{E (4\pi e^4 Z N_m) e} = \log_e \left(\frac{mc^2}{\epsilon_1} \right) \gamma \quad (5)$$

Where,

$\epsilon =$ Electron energy

To obtain the conditions, under which the electric field E is larger than $F_{\min}$, we introduce a dimensionless parameter, δ_0 such that:

$$\delta_0 = \frac{Emc^2}{4\pi e^3 Z N_m a}$$

$$a\delta_0 = \frac{Emc^2}{4\pi e^3 Z N_m} \quad (6)$$

Where e is the electron charge, a is the acceleration and c is the speed of light. Substituting (6) into (5) we have,

$$\frac{1}{Ee} \cdot a\delta_0 F = \log_e \left(\frac{mc^2}{\epsilon_1} \right) \gamma$$

Or,

$$\frac{a\delta_0 F}{Ee} = \log_e \left(\frac{mc^2}{\epsilon_1} \right) \gamma \quad (7)$$

Therefore,

$$F(v) = \frac{Ee}{a\delta_0} \log_e \left(\frac{mc^2}{\epsilon_1} \right) \gamma \quad (8)$$

but,

$$eE - F(v) = 0 \quad (9)$$

$$Ee = \frac{Ee}{a\delta_0} \log_e \left(\frac{mc^2}{\epsilon_1} \right) \gamma$$

Divide both sides by eE

$$1 = \frac{1}{a\delta_0} \log_e \left(\frac{mc^2}{\epsilon_1} \right) \gamma$$

$$a\delta_0 = \log_e \left(\frac{mc^2}{\epsilon_1} \right) \gamma \quad (10)$$

This implies that,

$$\left(\frac{mc^2}{\epsilon_1} \right) \gamma = \exp. a\delta_0 \quad (11)$$

Applying power series expansion of $e^{a\delta_0}$, we obtain

$$\left(\frac{mc^2}{\epsilon_1} \right) \gamma = e^{a\delta_0} \approx 1 + a\delta_0 + \frac{a^2 \delta_0^2}{2!} + \dots \quad (12)$$

$$\left(\frac{mc^2}{\epsilon_1} \right) \gamma = 1 + a\delta_0 \quad (13)$$

Where,

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

therefore,

$$\left(\frac{mc^2}{\epsilon_1}\right) = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx 1 + a\delta_o \quad (14)$$

So that,

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \frac{\epsilon_1}{mc^2} (1 + a\delta_o)$$

Or,

$$\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}} = \frac{\epsilon_1}{mc^2} (1 + a\delta_o) \quad (15)$$

$$\frac{mc^2}{\epsilon_1 (1 + a\delta_o)} = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad (16)$$

Squaring both sides;

$$\begin{aligned} \frac{m^2 c^4}{\epsilon_1^2 (1 + a\delta_o)^2} &= 1 - \frac{v^2}{c^2} \\ 1 - \frac{v^2}{c^2} &= \frac{m^2 c^4}{\epsilon_1^2} (1 + a\delta_o)^{-2} \end{aligned} \quad (17)$$

Using binomial expansion,

$$(1 + a\delta_o)^{-2} \approx (1 + (-2)a\delta_o + \dots) \quad (18)$$

Therefore,

$$1 - \frac{v^2}{c^2} \approx \frac{m^2 c^4}{\epsilon_1^2} (1 - 2a\delta_o)$$

So that,

$$\begin{aligned} \frac{v^2}{c^2} &\approx 1 - \frac{m^2 c^4}{\epsilon_1^2} + (1 - 2a\delta_o) \\ v^2 &\approx c^2 \left(1 - \frac{m^2 c^4}{\epsilon_1^2} + \frac{2m^2 c^4 a\delta_o}{\epsilon_1^2}\right) \\ v^2 &\approx c^2 - \frac{m^2 c^6}{\epsilon_1^2} + \frac{2m^2 c^6 a\delta_o}{\epsilon_1^2} \end{aligned} \quad (19)$$

Equation (19) shows that $v^2 \ll c^2$ which has an increased dynamical frictional force.

DISCUSSIONS

The cause of runaway electrons is assumed to be a uniform constant flux of cosmic ray secondaries in the earth's atmosphere. This occurs in the presence of electric field. According to Gurevich et al (1992), the electrons generated by cosmic rays are influenced by electric field of the earth's atmosphere. Since secondary cosmic ray electrons have high energies and move in all directions due to scattering, the acceleration can occur in any direction, depending on the sign of the electric field. Based on this, we choose to examine the rate at which these electrons accelerate within the non-relativistic region of the earth's atmosphere.

From the result of this work, the rate of acceleration of the runaway electrons in non-relativistic region was shown, using the equation of a cold electron in equation (1). We multiply the left hand side of equation (4) by E/E_0 . This gives us equation (5) which contains dimensionless parameters. Further decomposition of the equation, with the application of the force used by the electrons to accelerate in the atmosphere, we obtain equation (11). At this point we apply the power series expansion. The use of power series is to expand the equation, since it adds up to a known function. The power series expansion used is the e^x power series. This is because the defining property of e^x is that, it is equal to its own derivatives. In this work, e^x represents $e^{ax/b}$. This enables us to obtain equation (17). A close examination of equation (17), one will see that it is an algebraic that uses alphabet in place of figures. So we apply another mathematical tool known as binomial theorem. The obvious application of the binomial theorem is to expand algebraic expressions easily and conveniently. Other than that, the binomial theorem also helps us with simple numerical estimations. Owing to the fact that equation (17) deals with negative index, we therefore make use of the binomial theorem for negative index. Hence, we obtain equation (19). Equation (19) shows that an electron slows down when $v^2 \ll c^2$, which means there is an increase in the dynamical frictional force of the electron. Therefore, the cold electrons in the atmosphere undergo a dynamical frictional force and it plays a fundamental role in the study of electron acceleration mechanism. This confirms the idea of Milikh and Roussel (2010), that points out that, for runaway electron to accelerate in the presence of electric field the electric field strength E must exceed the critical value and the fast seed electrons with energies must be present. But equation (19) shows that the velocity of the electron is far less than the speed of light, which implies that the electron has very low velocity and hence may not accelerate.

CONCLUSION

Conclusively, since there is a decrease in the electron velocity in non-relativistic region of the earth's atmosphere, there is indeed a decrease in the electron activities. This means that there is absolutely a slowdown in some atmospheric processes within this region of the earth's atmosphere. Hence, spontaneous processes may not occur within the non-relativistic region of the earth's atmosphere.

REFERENCES

- [1] N. T. J. Alexander, Investigation of Runaway Electron Generation, Transport, and Stability in the DIII – 1 tokamak" page 4, **2011**.
- [2] M.L. Bell, F. Dominici and J. M. Samet, A meta-analysis of time-series studies of ozone and mortality with comparison to the national morbidity, mortality, and air pollution study. *Epidemiology*. **2005**;16(4): 436-45.
- [3] A.V.Gurevich, Runaway Electron Mechanism of Air Breakdown and Preconditioning During a Thunderstorm. *Phys. Lett. A*, **1992**;165: 463-468.
- [4] A.V. Gurevich, G. M. Milikh and R. Roussel Dupre. Runaway electron mechanism of air

breakdown and preconditioning during a thunderstorms, physics letters A, **1992**;165: 468.

[5] M. D. Kruskal and I. Bernstein. Princeton Plasma Physics Laboratory Report MATT -Q-20. Princeton University, **1962**.

[6] M. P.McCarthy, and G. K. Parks, On The Modulation of X-ray Fluxes in Thunderstorms. *J. Geophys. Res.* **1992**;97: 5857.

[7] G. Milikh, and R. Roussel-Dupre, Runaway Breakdown and Electrical Discharges in Thunderstorms, *J. Geophys. Res.*, **2010**;115: pp. 1-3.

[8] K. Molvig, and M. S. Tekula, "Theory of runaway electrons" pg 27, **1977**.G. L. Nicolai, (**2000**), Relativistic Runaway Electrons Above Thunderstorms. PhD thesis; Stanford University.

[9] Runaway SADF Space Astronomy Discusion Forum. Lightning and The Breakdown Theory SAD Forum. P. I, **2009**.

[10] H. Tsuchiya, T. Enoto, S. Yamada, T. Yuasa, M. Kawaharada, T. Kitaguchi, M. Kokubun, H. Kato, M. Okano, S. Nakamura, and K. Makishima, Detection of High-energy Gamma Rays From Winter Thunderclouds. *Phys. Rev.Lett.*, **2007**;99: 165002.

[11] V. M. Vasyliunas, Upper Limit on The Electric Field Along a Magnetic Line.*J.Geophys. Res.*, vol. 85 p.4616-4620. Viggiano, A. and Arnold, F. Ion Chemistry and Composition of The Atmosphere. Handbook of Atmospheric electrodynamics, **1980**;1: 1-26.